RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2017

FIRST YEAR [BATCH 2017-20]

MATHEMATICS (General)

Paper : I

Date : 23/12/2017

Time : 11 am – 2 pm

Full Marks : 75

(Use a separate Answer book for each group)

Group – A

Answer **any five** of the following: 5×5

1. a) Express $z = \left(\sqrt{3}-1\right) + i\left(\sqrt{3}+1\right)$ in $re^{i\theta}$ form, and find the values of $z^{1/4}$. 3

   b) Find the modulus and the principal amplitude of $\dfrac{(2+3i)^2}{2+i}$. 2

2. Show that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$. 5

3. If α, β, γ be the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $(\beta-\gamma)^2$, $(\gamma-\alpha)^2$, $(\alpha-\beta)^2$. 5

4. Solve the system of equations using Cramer's rule: 5

$$x + 2y + 3z = 5$$
$$3x - 2y + z = -1$$
$$4x + 2y + z = 13$$

5. Find the rank of the matrix $A = \begin{bmatrix} -8 & 7 & -10 \\ 48 & -42 & 60 \\ 40 & -35 & 50 \end{bmatrix}$. 5

6. a) State the Descarte's rule of sign. 2

   b) Apply the Descarte's rule of sign and Rolle's theorem to find the number of real roots and complex roots of the polynomial equation: $x^7 - 3x^3 - x + 1 = 0$. 3

7. Solve by Cardan's method $x^3 - 27x - 54 = 0$. 5

8. Investigate for what values of a and b, the simultaneous equations

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + az = b$$

have (i) no solution and (ii) a unique solution. 3+2

Group – B

Answer **any five** of the following: 5×5

9. Prove that the set of all 2x2 matrices with real entries forms a vector space over (real numbers) with respect to matrix addition and multiplication of matrices by constants. 5

10. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, then use Cayley-Hamilton theorem to express $\left(A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2\right)$ as a linear polynomial in A. 5

11. Show that the vectors (1, 2, 1), (2,1, 0), (1, –1, 2) form a basis of the vector space ^{3} over the field of real numbers. 5

12. Let $Z = \{0, \pm1, \pm2, \pm3, \cdots\}$. The binary operation $*$ on Z is defined by $a * b = a - b\,(a, b \in Z)$,
    i) Find $x \in Z$, such that $x * 3 = 3$,
    ii) Show that the operation $*$ is neither commutative nor associative. 1+4

13. Examine whether the quadratic form $5x^2 + y^2 + 5z^2 + 4xy - 8xz - 4yz$ is positive definite or not. 5

14. i) Give an example of a function $f : \mathrm{R} \to \mathrm{R}$ which is injective but not surjective. 2
    ii) If $f : A \to B$ and $g : B \to C$ are both surjective, then show that $g \circ f : A \to C$ is also surjective. 3

15. Show that $(m\mathrm{Z}, +)$ forms a subgroup of $(\mathrm{Z}, +)$ for all $m \in \mathrm{Z}$. 5

16. If $(R, +, \cdot)$ be a ring and $a, b \in R$, then show that (i) $a \cdot (-b) = (-a) \cdot b$, (ii) $(-a) \cdot (-b) = a \cdot b$. 2+3

Group – C

Answer **any five** of the following: 5×5

17. Find y_n, if $y = x^3 \log_e x$. 5

18. Show that the sum of intercepts on the axes of any tangent to the curve $\sqrt{x} + \sqrt{y} = 3$ is a constant. 5

19. Verify Euler's theorem for $f(x, y, z) = 3x^2yz + 5xy^2z + 5z^4$. 5

20. Find the pedal equation of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ with respect to the origin. 5

21. $y = \left(\sin^{-1} x\right)^2$ prove that $\left(1 - x^2\right) y_{n+2} - (2n+1)\, x y_{n+1} - n^2 y_n = 0$. 5

22. In the case of folium of Descartes $x^3 + y^3 = 3axy$, prove that the radius of curvature at $\left(3a/2, 3a/2\right)$ is numerically equal to $3\sqrt{2}\, a/16$. 5

23. If $u = f\left(x^2 + 2yz, y^2 + 2zx\right)$, prove that $\left(y^2 - zx\right)\frac{\partial u}{\partial x} + \left(x^2 - yz\right)\frac{\partial u}{\partial y} + \left(z^2 - xy\right)\frac{\partial u}{\partial z} = 0$. 5

24. 5

$$\begin{aligned} f(x, y) &= xy\frac{x^2 - y^2}{x^2 + y^2}, && (x, y) \neq (0,0) \\ &= 0 && , (x, y) = (0,0) \end{aligned}$$

Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

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